Uniform rates of the Glivenko-Cantelli convergence and their use in approximating Bayesian inferences

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Abstract

This talk deals with suitable quantifications in approximating a probability measure by an "empirical" random probability measure $\hat{\mathfrak{p}}_n$, depending on the first n terms of a sequence $\{\tilde{\xi}_i\}_{i\geq 1}$ of random elements. In the first part, we study the range of oscillation near zero of the p-Wasserstein distance $\mathrm{d}^{(p)}_{[\mathbb{S}]}$ between $\mathfrak{p}_0$ and $\hat{\mathfrak{p}}_n$, assuming the $\tilde{\xi}_i$'s i.i.d. from $\mathfrak{p}_0$. Our first main result deals with the so-called nonparametric model, in which $\mathfrak{p}_0$ is fixed in the space of all probability measures on $(\mathbb{R}^d, \mathscr{B}(\mathbb{R}^d))$ and $\hat{\mathfrak{p}}_n$ coincides with the empirical measure $\tilde{\mathfrak{e}}_n := \frac{1}{n}\sum_{i=1}^{n} \delta_{\tilde{\xi}_i}$. The second (third, respectively) result considers the case in which $\mathfrak{p}_0$ is a d-dimensional Gaussian distribution (an element of a distinguished statistical exponential family, respectively) and $\hat{\mathfrak{p}}_n$ is another d-dimensional Gaussian distribution with estimated mean and covariance matrix (another element of the same family with estimated parameter, respectively). These new results improve on allied recent works by providing *uniform bounds* with respect to n, meaning that the finiteness of $\mathsf{E}\Big[\Big(\sup_{n\geq 1} b_n \mathrm{d}^{(p)}_{[\mathbb{S}]}(\mathfrak{p}_0, \hat{\mathfrak{p}}_n)\Big)^p\Big]$ is proved for some diverging sequence b_n of positive numbers. In the second part, assuming the $\tilde{\xi}_i$'s exchangeable, we study the range of oscillation near zero of the Wasserstein distance between the conditional distribution—also called posterior—of the directing measure of the sequence, given $\tilde{\xi}_1, \ldots, \tilde{\xi}_n$, and the point mass at $\hat{\mathfrak{p}}_n$. Similarly, a bound for the approximation of predictive distributions is given. Finally, by exploiting the uniform bounds given in the first part, the results about the nonparametric, Gaussian and exponential models are reconsidered and reformulated according to a Bayesian perspective.